

Homework problem set 4 Math 423/502

1. Given a short exact sequence of complexes

$$0 \rightarrow F'_\bullet \xrightarrow{\alpha} F_\bullet \xrightarrow{\beta} F''_\bullet \rightarrow 0$$

define the connecting homomorphism $\delta_k : H_k(F''_\bullet) \rightarrow H_{k-1}(F'_\bullet)$. Prove that the construction is independent of the choices.

2. Prove that the corresponding long sequence is exact:

$$\cdots \xrightarrow{\delta_{k+1}} H_k(F'_\bullet) \xrightarrow{\alpha_k} H_k(F_\bullet) \xrightarrow{\beta_k} H_k(F''_\bullet) \xrightarrow{\delta_k} H_{k-1}(F'_\bullet) \rightarrow \cdots$$

3. Prove the Snake lemma (see exercise A3.10 in Eisenbud)
4. Prove the 5-lemma (see exercise A3.11 in Eisenbud)
5. Prove the 9-lemma (see exercise A3.12 in Eisenbud)