



all columns exact

$$T_x = \{ m \in M : xm = 0 \}$$

Now the usual "staircase" diagram chase constructs the isomorphism

$$\text{Tor}_1 \left(\frac{R}{(x)}, M \right) = \frac{\text{ker} \left(\frac{R}{(x)}^b \rightarrow \frac{R}{(x)}^a \right)}{\text{Im} \left(\frac{R}{(x)}^c \rightarrow \frac{R}{(x)}^a \right)} \xrightarrow{\alpha} T_x \xrightarrow{\eta}$$