1. **Semi-simple TQFTs.** A TQFT/Frobenius algebra $Z(-)/A$ is called semi-simple if $A$ is semi-simple: i.e. there exists a basis $e_1, \ldots, e_n$ of $A$ such that $e_i \cdot e_j = \delta_{ij} e_i$. Let $\mu_i = \mu(e_i)$ where $\mu$ is the co-unit.
   
   (a) Show that $\mu_i \neq 0$.
   
   (b) Define $\lambda_i = \mu_i^{-1}$. Show that the value of the TQFT on the closed, genus $g$ surface is given by
   
   $$Z(g) = \sum_{i=1}^{n} \lambda_i^{g-1}.$$  

   Hint: compute the “genus adding operator” $G : A \to A$ given by evaluating $Z$ on the genus 1 cobordism from the circle to the circle by observing it is the composition of the coproduct with the product.

2. **Genus adding operator in various cases.** Given a TQFT/Frobenius algebra $Z(-)/A$, let $G : A \to A$ be the genus adding operator defined in the hint above. Let $Z(g)$ be the value of the TQFT on the closed genus $g$ surface.
   
   (a) Let $A = \mathbb{C}[x]/(x^2 + 1)$, having co-unit $\mu(a + bx) = a$. Compute $G : A \to A$ (as a matrix with respect to the basis $\{1, x\}$) and $Z(g)$.
   
   (b) Let $A = \mathbb{C}[x]/(x^2 - 1)$, having co-unit $\mu(a + bx) = a$. Compute $G : A \to A$ (as a matrix with respect to the basis $\{1, x\}$) and $Z(g)$.
   
   (c) Let $A = \mathbb{C}[x]/(x^2)$, having co-unit $\mu(a + bx) = b$. Compute $G : A \to A$ (as a matrix with respect to the basis $\{1, x\}$) and $Z(g)$.
   
   (d) (Some knowledge of topology required). Let $A = H^{ev}(M, \mathbb{C})$ be the even degree part of the cohomology of a compact oriented even dimensional manifold $M$. $A$ is a commutative algebra via cup product and has co-unit given by $\mu : A \to \mathbb{C}$ where $\mu(\omega) = \int_M \omega$ (in particular $\mu(\omega) = 0$ unless $\text{deg}(\omega) = \dim(M)$). Note that Poincaré duality implies that the associated bilinear form is non-degenerate. Compute $G : A \to A$ and $Z(g)$ in this case.
   
   (e) Which of the above cases are semi-simple?

3. **The Fibonacci TQFT.** Let $A = \mathbb{C}[x]/(x^2 - x - 1)$ with $\mu(1) = -1$ and $\mu(x) = 2$. Compute the $G$, the genus adding operator, as a matrix with respect to the basis $\{1, x\}$. Let $W^1(g)$ be the genus $g$ cobordism from the empty set to the circle. Show that $Z(W^1(g))$ viewed as an element of $A$ is given by
   
   $$f^n_{g-1} + f_g x$$

   where $f_n$ is the $n$th Fibonacci number.